Seat No. : _______________

N14-117

November-2014

B.Sc., Sem.-V

MAT : 302 – Mathematics

(Analysis-1)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory.

(2) Figures to the right indicates full marks of the question.

1. (a) Prove that there exists a real number x such that $x^2 = 2$. **7**

OR

If In = [an, bn], n $\in$ N is a nested sequence of a closed bounded intervals then prove that there exists a number $\xi \in$ R such that $\xi \in I_n$ for all n $\in$ N.

(b) State and prove Archimedian Property. Using this prove that if S = {1/n : n $\in$ N}, then inf S = 0. **7**

OR

Prove that the set Q of all rational number is denumerable.

2. (a) State and prove Bolzano-Weierstrass theorem. **7**

OR

Prove that a real sequence is Cauchy if and only if it is convergent.

(b) State and prove squeeze theorem, using this prove that for p $\geq$ 2 $\lim_{n \to \infty} \frac{1}{n^p} = 0$. **7**

OR

If $\{x_n\}$ converges to the value a, then prove that every subsequence of this sequence converges to a.

3. (a) State and prove Intermediate Value Theorem. **7**

OR

Suppose that $\lim_{x \to c} f(x) = L_1$ and $\lim_{x \to c} g(x) = L_2$. Then prove that

(i) $\lim_{x \to c} f(x)\, g(x) = L_1\, L_2$

(ii) If $L_2 \neq 0$, then $\lim_{x \to c} \frac{(x)}{g(x)} = \frac{L_1}{L_2}$

(b) Discuss the uniform convergence of the following functions : **7**

(i) $f(x) = 1/x$ on $[c, \infty)$, $c > 0$

(ii) $g(x) = \sin(2\pi/x)$ on $(0, 1)$

OR

Suppose that the function f is continuous on the interval [a, b] then prove that f is uniformly continuous on [a, b].

4. (a) State and prove chain rule. **7**

OR

State and prove Mean Value Theorem and verify for the function

$f(x) = \sqrt{x} - x$ on [0, 4]

(b) State and prove L'Hospital's First Rule. **7**

OR

Suppose that f is continuous and one-to-one on [a, b]. Then prove that f^{-1} is continuous on f([a, b]).

5. Answer the following questions : **14**

(1) Define countable and uncountable sets.

(2) Give two binary representations of 3/8.

(3) Show that $\lim_{n \to \infty} \ln(1/n) = -\infty$

(4) Find the $\lim_{x \to 0} \frac{x}{|x|}$ if exists.

(5) Give example of function which is nowhere continuous.

(6) Find the cluster points of the sequence $\{x_n\} = \{\sin(n\pi/4)\}$.

(7) Evaluate $\lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^x$